

Calculus Cheat Sheet

Derivatives

Definition and Notation

If _____ then the derivative is defined to be $\lim_{h \rightarrow 0} \frac{\text{_____}}{\text{_____}}$.

If _____ then all of the following are equivalent notations for the derivative.

— — —

If _____ all of the following are equivalent notations for derivative evaluated at _____.

| —| —|

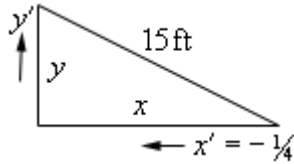
Interpretation of the Derivative

If _____ then,

Related Rates

Sketch picture and identify known/unknown quantities. Write down equation relating quantities and differentiate with respect to using implicit differentiation (add on a derivative every time you differentiate a function of). Plug in known quantities and solve for the unknown quantity.

Ex. A 15 foot ladder is resting against a wall. The bottom is initially 10 ft away and is being pushed towards the wall at $\frac{1}{4}$ ft/sec. How fast is the top moving after 12 sec?



x' is negative because x is decreasing. Using Pythagorean Theorem and differentiating,

$$y^2 + x^2 = 15^2 \quad 2y \frac{dy}{dt} + 2x \frac{dx}{dt} = 0$$

After 12 sec we have $x = 10 - 12 \cdot \frac{1}{4} = 7$ and

so $y = \sqrt{15^2 - 7^2} = \sqrt{176}$. Plug in and solve for $\frac{dy}{dt}$.

$$7 \cdot \frac{1}{4} + \sqrt{176} \frac{dy}{dt} = 0 \quad \frac{dy}{dt} = -\frac{7}{4\sqrt{176}} \text{ ft/sec}$$

Ex. Two people are 50 ft apart when one starts walking north. The angle θ changes at 0.048 rad/sec . At $t = 10$ sec, how fast is the distance between them changing when $\theta = 0.6$ rad?

Integrals Definitions

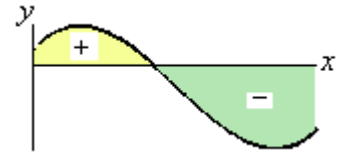
Definite Integral: Suppose f is continuous on $[a, b]$. Divide $[a, b]$ into n subintervals of width Δx and choose x_i^* from each interval.

Then $\lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i^*) \Delta x = \int_a^b f(x) dx$.

Calculus Cheat Sheet

Applications of Integrals

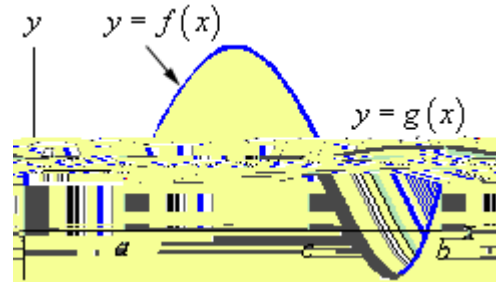
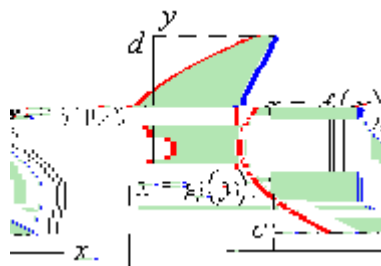
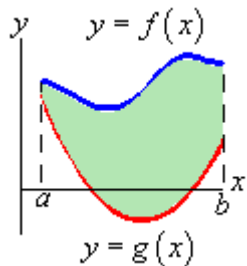
Net Area : represents the net area between and the x -axis with area above x -axis positive and area below x -axis negative.



Area Between Curves : The general formulas for the two main cases for each are,

$\int_a^b (\text{upper function}(x) - \text{lower function}(x)) dx$ & $\int_c^d (\text{right function}(y) - \text{left function}(y)) dy$

If the curves intersect then the area of each portion must be found individually. Here are some sketches of a couple possible situations and formulas for a couple of possible cases.



Volumes of Revolution : The two main formulas are $\int_a^b 2\pi x (f(x) - g(x)) dx$ and $\int_c^d 2\pi y (g(y) - f(y)) dy$. Here is some general information about each method of computing and some examples.

Rings

$2\pi \int_a^b (x^2 - y^2) dx$

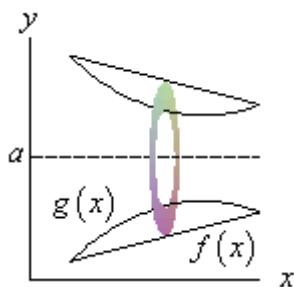
Limits: \int_a^b of right/bot ring to \int_c^d of left/top ring
 Horz. Axis use $\int_a^b 2\pi x (f(x) - g(x)) dx$, Vert. Axis use $\int_c^d 2\pi y (g(y) - f(y)) dy$,
 and $\int_a^b 2\pi x (f(x) - g(x)) dx$, and $\int_c^d 2\pi y (g(y) - f(y)) dy$.

Cylinders

$2\pi \int_a^b r^2 dy$

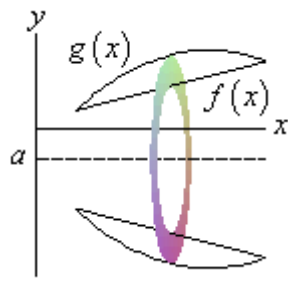
Limits: \int_c^d of inner cyl. to \int_a^b of outer cyl.
 Horz. Axis use $\int_c^d 2\pi y (g(y) - f(y)) dy$, Vert. Axis use $\int_a^b 2\pi x (f(x) - g(x)) dx$,
 and $\int_c^d 2\pi y (g(y) - f(y)) dy$, and $\int_a^b 2\pi x (f(x) - g(x)) dx$.

Ex. Axis : $x=0$



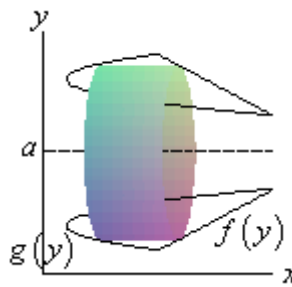
outer radius : $|x|$
 inner radius : $|y|$

Ex. Axis : $x=0$



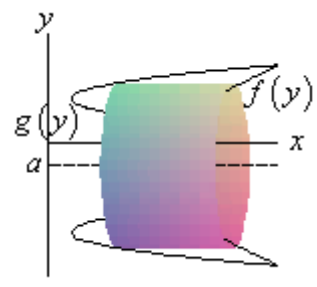
outer radius: $|x|$
 inner radius: $|y|$

Ex. Axis : $x=0$



radius : $|y|$
 width : $|x|$

Ex. Axis : $x=0$



radius : $|y|$
 width : $|x|$

These are only a few cases for horizontal axis of rotation. If axis of rotation is the x -axis use the $\int_a^b 2\pi y (f(y) - g(y)) dy$ case with $x=0$. For vertical axis of rotation ($x=c$ and $x=d$) interchange x and y to get appropriate formulas.

