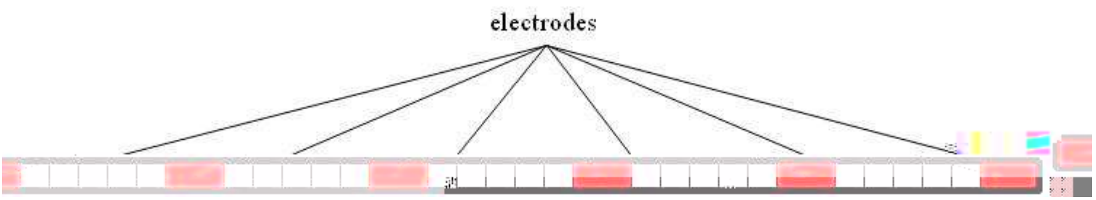
Solving the Bioheat Equation for

Electrophysiology

- The study of the body's electric activity
  - Can be small-scale (individual cells) or large-scale (entire organs)
- Electrophysiology often plays an important role in medical diagnostic procedures
  - Ex: ECG, EEG, EMG
- Signals often recorded by placing a series of electrodes on the surface of a patient's skin
  - Not always a practical approach—long-term collection of data may be required

A Possible Solution

- A subcutaneous (under the skin) recording device could remain in place semi-permanently
  - Device may be implanted almost anywhere in a minor surgical procedure



Schematic of proposed device

Figure created by Zachary Abzug

Transcutaneous Recharging

- Most implanted devices recharged via magnetic fields—not feasible for this device
- Instead, induce high frequency electric field using external source and sink electrodes

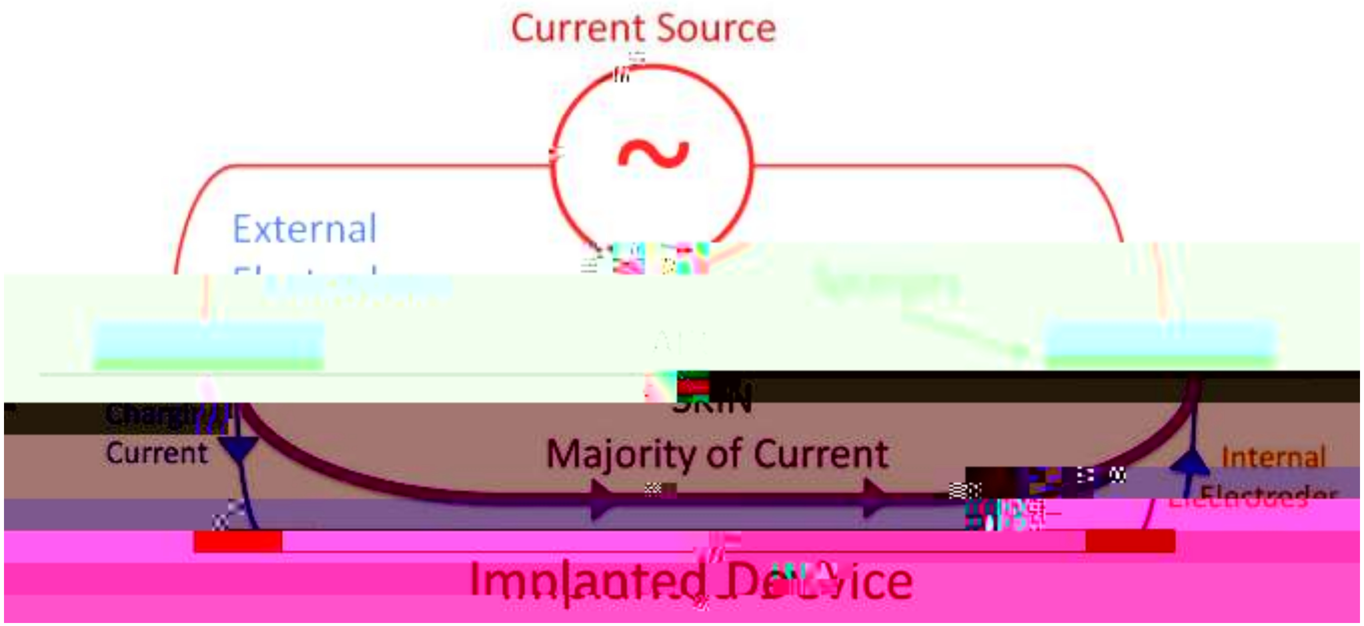


Figure created by Zachary Abzug

Project Objective

- Derive a closed-form solution for the anticipated temperature increase
  - Primary motivation: improved understanding of physical parameters on temperature increase

The “Extended” Bioheat Equation

$$\rho C \frac{\partial T}{\partial t} = k\nabla^2 T + \rho_b C_b \omega_b (T_b - T) + Q_{met} + \boldsymbol{J} \cdot \boldsymbol{E}$$

- The heat equation: describes variation of

Power Dissipation

- Calculate the work done by electromagnetic forces on a charge Q moving some infinitesimal distance d :
-

Simplifying the Bioheat Equation

- Must make several simplifications

$$\rho C \frac{\partial T}{\partial t} = k\nabla^2 T + \rho_b C_b \omega_b (T_b - T) + Q_{met} + \boldsymbol{J} \cdot \boldsymbol{E}$$

Steady-state solution( )

Ignore perfusion( )

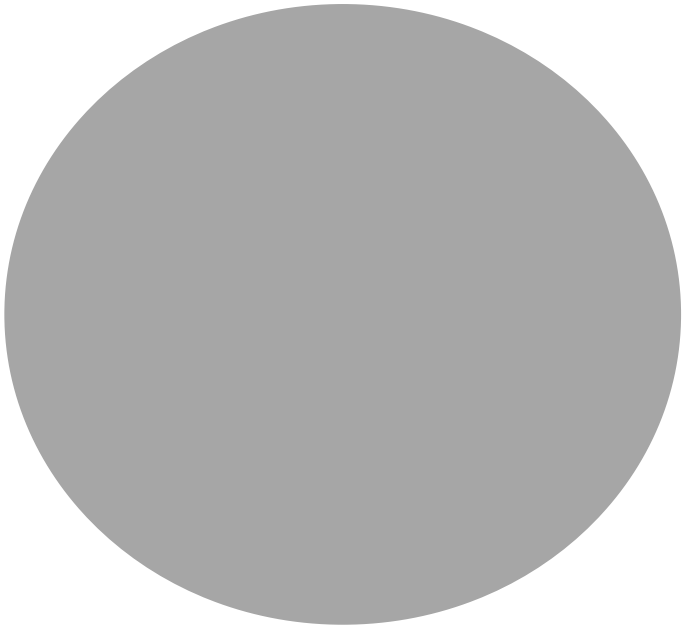
Ignore metabolic heat production( )

- Final equation to solve:

A Previous Solution

Geometric Considerations

- Treat electrode as a current-producing sphere in an infinite homogeneous and isotropic resistive material
- Second electrode is at infinity (V=0)



A Solution in Spherical Coordinates

- Write bioheat equation in spherical coordinates
  - Ignore θ and φ . u

Solving Laplace's Equation

- To determine A, consider a point source of current in an infinite, homogeneous, isotropic medium.
  - The current density is: $\mathbf{J} = \frac{I}{4\pi r^2}\hat{\mathbf{r}}$
  - Since $\mathbf{J} = \sigma\mathbf{E} = -\sigma\nabla V$, the potential is:

$$\nabla V = -\frac{I}{4\pi\sigma r^2}\hat{\mathbf{r}} \Rightarrow \frac{dV}{dr} = -\frac{I}{4\pi\sigma r^2} \Rightarrow V(r) = \frac{I}{4\pi\sigma r}$$

  - Compare to $V(r) = -\frac{A}{r} \Rightarrow A = -\frac{I}{4\pi\sigma}$

Solving the Bioheat Equation

The Particular Solution

- To use variation of parameters, rewrite as:

$$y'' + \frac{2}{x}y' = \frac{c}{x^4}$$

- The solution is given by $y_p = u_1 y_1 + u_2 y_2$, where y_1 and y_2 are from the complimentary function and

$$u_1 = \int u_1' = \int \frac{W_1}{W} \qquad u_2 = \int u_2' = \int \frac{W_2}{W} \qquad W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$W_1 = \begin{vmatrix} 0 & y_2 \\ f(x) & y_2' \end{vmatrix} \qquad W_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & f(x) \end{vmatrix} \qquad f(x) = \frac{c}{x^4}$$

$$y_p = \frac{c}{2x^2}$$

The General Solution

- The general solution is the sum of the complimentary function and the particular solution:

$$y = y_c + y_p = \alpha + \frac{\beta}{x} + \frac{c}{2x^2}$$

$$T(r) = \alpha + \frac{\beta}{r} + \frac{c}{2r^2}$$

$$\boxed{T(r) = \alpha + \frac{\beta}{r} - \left(\frac{I}{4\pi}\right)^2 \frac{1}{2k\sigma r^2}}$$

- Plugging this back into the bioheat equation verifies that it is a solution

The General Solution

- The solution is also valid in terms of units

Quantity	Unit
I	A
k	A/V*m
ů	V*A/m*K
Ǔ	K
ɓ	K*m

- Need to determine α and β

Determining α

- Assume the tissue is unaffected by heating at an

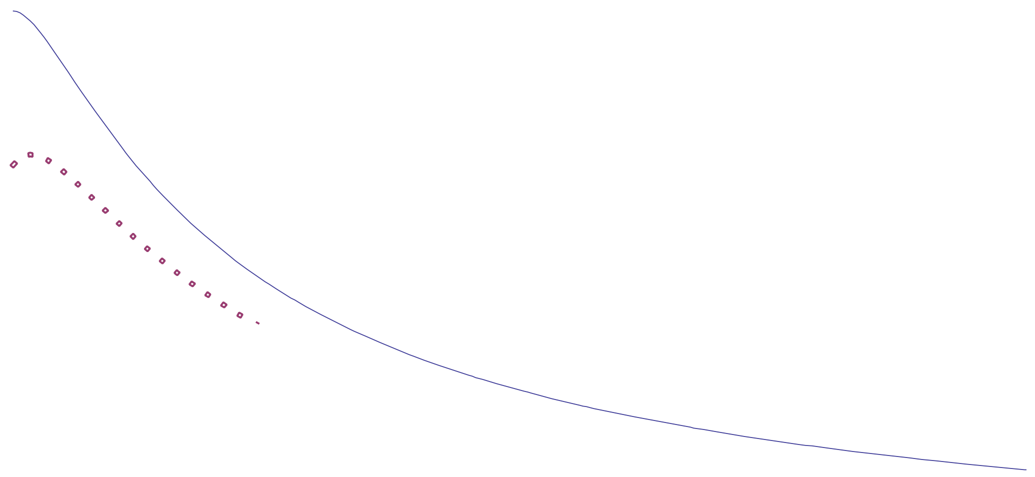
Determining β

- The solution for temperature is:

A plot of temperature vs. radial distance from electrode (I = 11.7 mA, σ = 0.327 A/V*m, k = 0.565 W/m*K, r_0 = 0.635 mm).

Sensitivity to r_0

- Behavior of solution is highly dependent on r_0



Dependence of temperature on r_0

Future Work

- What is the *physical* meaning of the solution?
- The temperature distribution is

or

$$T(r) = 310.15 + \frac{A}{r_0 r} - \frac{A}{2r^2}$$ (where $A = \left(\frac{I}{4\pi}\right)^2 \frac{1}{k\sigma}$)

- What does it mean to have two similar terms competing?

Conclusions

- Recharging a subcutaneous medical device using electric fields can increase tissue temperature
- We show that the steady-state temperature distribution is given by $T(r) = 310.15 + \frac{\beta}{r} - \left(\frac{I}{4\pi}\right)^2 \frac{1}{2k\sigma r^2}$
- Future work: investigate how physical parameters influence temperature increase

References

[1] Elwassif, Maged M., Qingjun Kong, Maribel Vazquez, and Marom Bikson, “Bio-

Questions?

Extra Slides

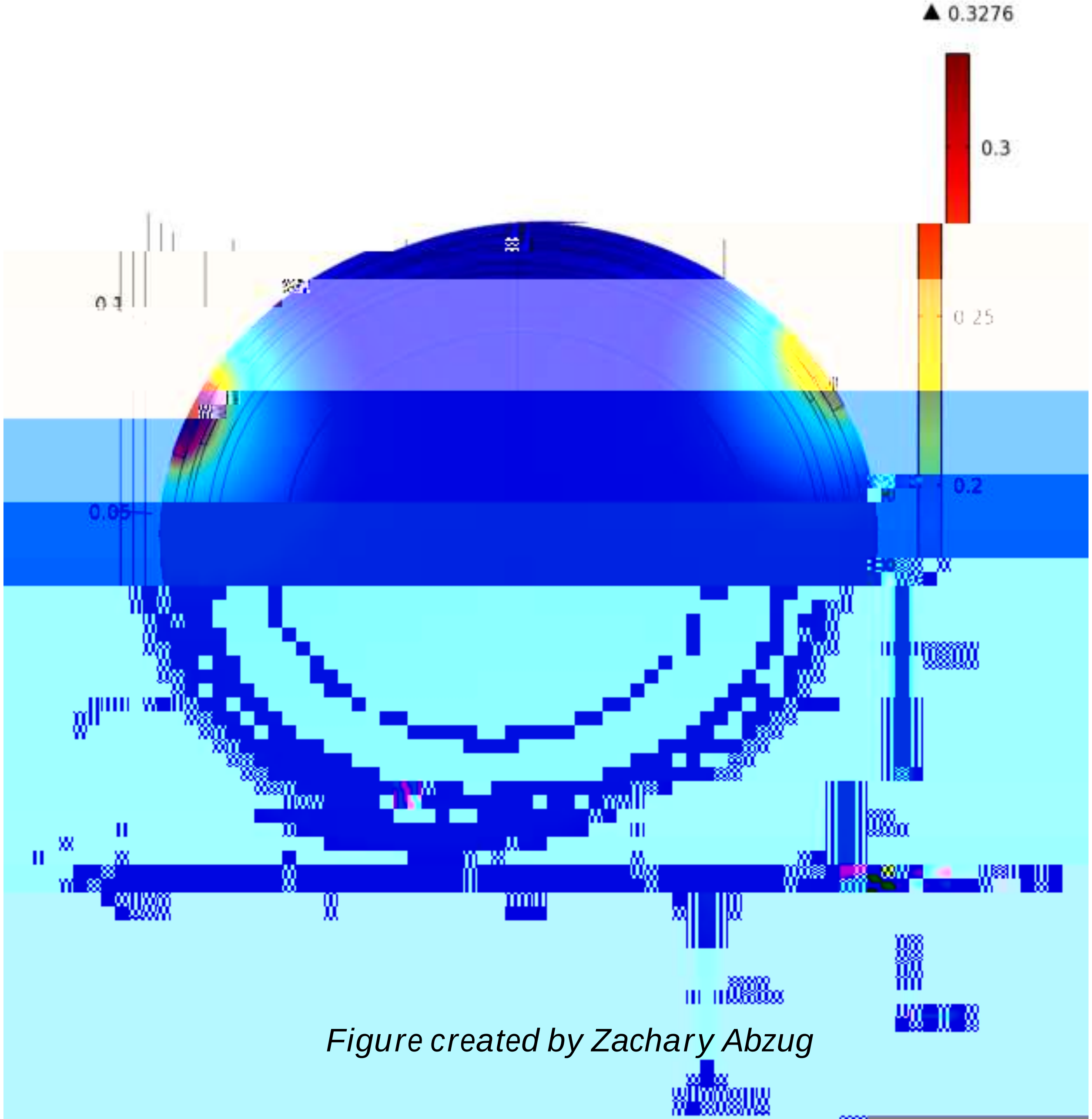


Figure created by Zachary Abzug

The Heat Equation

- The heat equation describes the three-dimensional variation of temperature in a region as a function of time

$$\rho C \frac{\partial T}{\partial t} = k \nabla^2 T$$

ρ = density
C = specific heat
k = thermal conductivity

- Not a complete model of heat transfer in biological situations due to perfusion (blood flow)

The Bioheat Equation

- The rate of heat transfer between blood and tissue is proportional to:
  - The volumetric perfusion rate
  - The difference between the arterial blood temperature and the local temperature
- Also add term (Q_{met}) to account for metabolic heat production
- The bioheat equation is:

$$\rho C \frac{\partial T}{\partial t} = k\nabla^2 T + \rho_b C_b \omega_b (T_b - T) + Q_{met}$$

ρ_b = density of blood

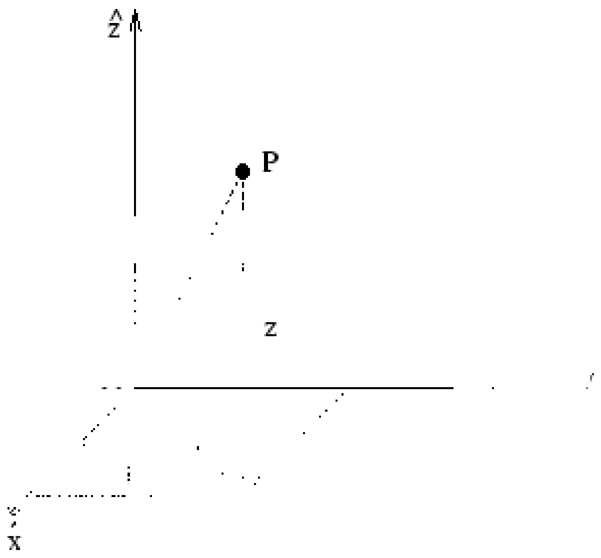
C_b = specific heat of blood

T_b = temperature of blood

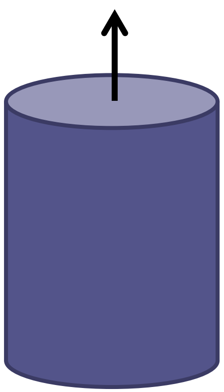
ω_b = perfusion rate per unit volume of tissue

T = local tissue temperature

A Solution in Cylindrical Coordinates



Cylindrical coordinate system
(image from uic.edu)

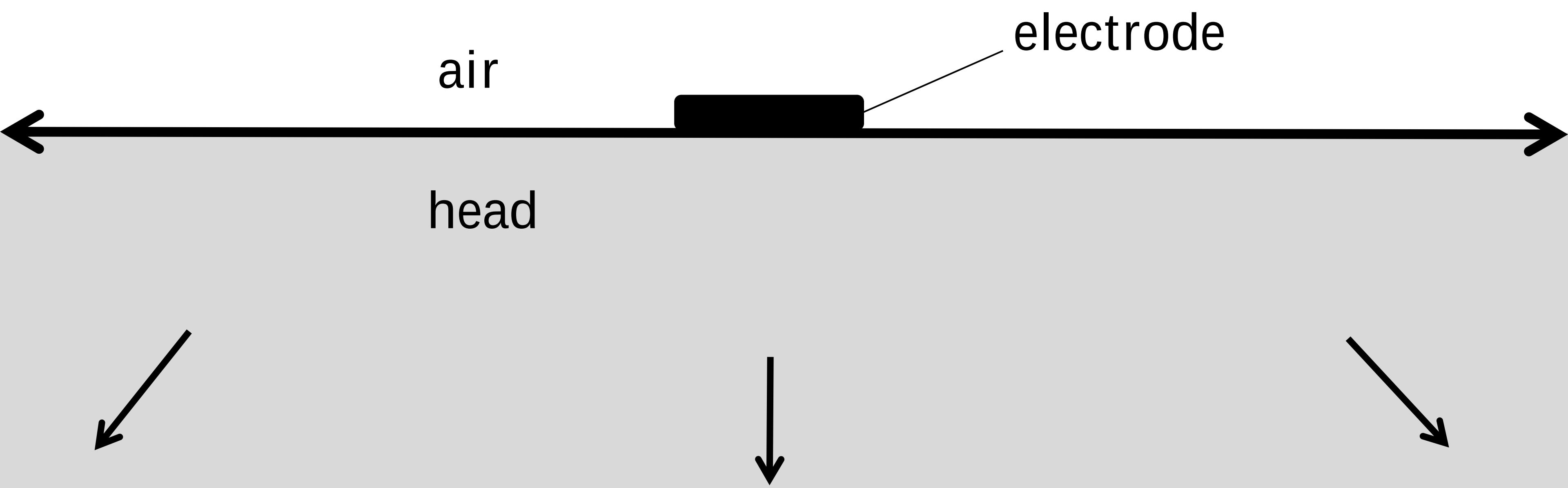


- Wrote Laplacian in cylindrical coordinates, ignoring φ and z dependence due to geometry

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = -\frac{\sigma}{k}|\nabla V|^2$$

A Solution in Cylindrical Coordinates

- For our analysis, model head as infinitely wide and deep homogeneous resistive material
- Place one electrode on surface ($V=V_{applied}$) and one electrode at infinity ($V=0$)



- It’s acceptable to ignore φ dependence in our situation because of the axial symmetry
- Problem: we cannot ignore *z* dependence

Determining β – Approach #1

- Want to know how the temperature behaves at

Determining β – Approach #1

A plot of temperature vs. radial distance from electrode (I = 11.7 mA, σ = 0.327 A/V*m, k = 0.565 W/m*K, r_0 = 0.635 mm).

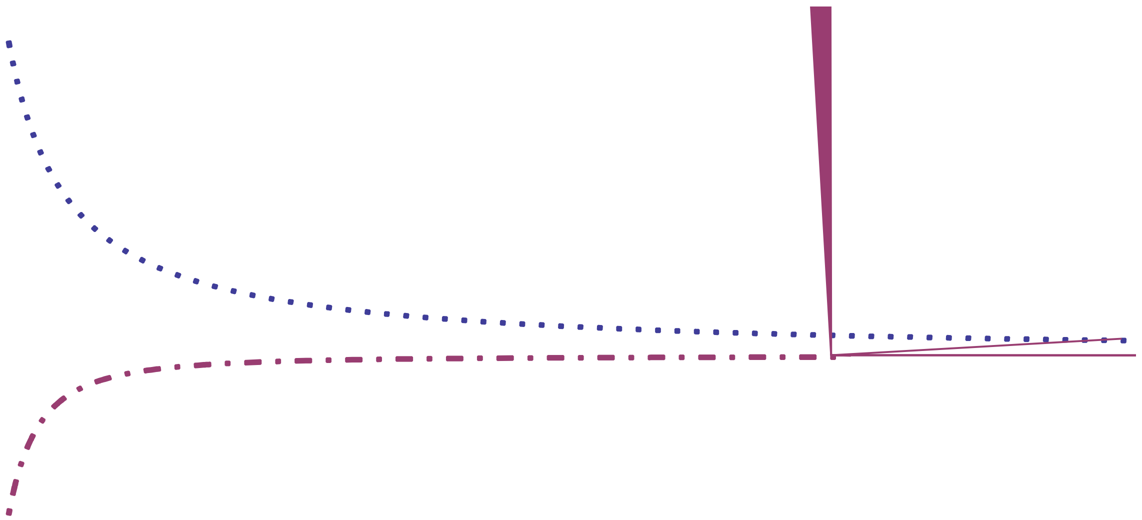
Determining β – Approach #1

A plot of temperature vs. radial distance from electrode (I = 11.7 mA, σ = 0.327 A/V*m, k = 0.565 W/m*K, r_0 = 0.635 mm).

Temperature Peak

Temperature Peak

- Would like to account for the peak in temperature
- Remember solution is of the form: $T(r) = \alpha + \frac{\beta}{r} + \frac{c}{2r^2}$



A plot of comparing the contribution of the β/r and c/2r² terms